The mathematical logic of narrative[1]

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In recent years, mathematics is increasingly becoming a subject for the narrative arts, mathematicians and the theorems they are trying to prove appearing in starring parts in novels, plays and films. This is a welcome development, carrying with it the hope that the queen of the sciences is now, at long last, entering the culture at large, from which it had always been regally isolated.

Yet, it is not about the stories of mathematics I want to talk today, but about the mathematics of stories.

"The mathematics of *stories*?" -- I can almost hear the question. For indeed, unless a story is of the type "Mary bought twelve pieces of candy and wants to divide it equally between her three friends", what, in Euclid's name, *are* its mathematics? Well, let me say it from the start: nobody knows. I do not plan not present you with a science. I only want to say a few things about the directions in which some people are moving, to arrive one day, possibly, at an understanding of aspects of storytelling that can be called, partly, mathematical.

For unlike physics, the study of physical nature, where mathematics plays a dominant role, in the study of storytelling – *narratology* is the grander name -- mathematics does not appear. Storytelling, after all, is an art. Yet, painting is also an art, but mathematics plays a crucial part in it, the discovery of the geometric laws of perspective having helped to usher in a whole new era. And of course storytelling, like painting, is not *just* an art. Any of its adept practitioners will tell you that there is only so much that they can do relying solely on feelings and inspiration, without logic and technique, i.e. the very things that guide progress in any scientific domain. As an early Renaissance

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master-builder said to his masons: *ars sine scientia nihil*. Art without science, is nothing.

And science often relies heavily on mathematics. Which brings us straight to our subject.

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Before we talk about the possible mathematization of narratology, let us look for a moment at applying mathematics in the general case, the very process of mathematization.

There are some basic truths to remind ourselves of and the first is that the mathematization of an extra-mathematical field should not be an end in itself -- not ever. Mathematization is a tool, a method that is useful only to the extent that it promotes advancement of the field itself. We do not mathematize a science just for the fun of it – although fun may be a fine personal motive for some mathematizers! – but to increase, through mathematical means, our understanding of its inherent truths. And though, of course, mathematization has often helped mathematics by providing internal challenges and problems – the mathematization of physics very famously so – it is not this dimension that concerns us but the opposite: how much mathematization may help the subject. If it does, fine. If not, drop it.

The second truth, is that mathematization of a science, to be of some use, should come rather late in the science's development, i.e. after it has acquired a body of knowledge it can call its own. And, in fact, the cases where this is not the case, where the onslaught of mathematizers on an unsuspecting body of knowledge comes too soon in, are the exceptions that prove the rule: for in these cases, the results of mathematization for the subject are usually trivial; much good mathematics, possibly, but little good sense.

A good case in point is so-called "Catastrophe Theory" – I will refrain from facile jokes, about what eventually happened to it – created in the nineteen sixties by the mathematician René Thom to explain, among other things, phenomena in biology. (In fact, Catastrophe Theory captured the popular imagination back then somewhat in the way that Chaos Theory did

more recently – though not to such an extent – as a mathematical magical tool, a potential Theory of Everything.) But Catastrophe Theory didn't deliver: the mathematical baggage was too heavy and the realities it purported to throw light on could not support it. Too much math is often as bad, or even worse, as too little. At least, when we are using too little we can become aware of the need for more. But when there is too much of the glittery stuff, it becomes like the Emperor's new clothes. Zero posing for treasure.

Which brings us, naturally, to the third truth about mathematization, the reason why it can fail even when the time for it is epistemologically propitious: mathematization should not be done by mathematicians, anyway not by mathematicians alone – expert knowledge of the field is also needed. And even if the person attempting the mathematization is a great mathematician – and the father of Catastrophe Theory was just that, he even had a Fields Medal to prove it! --, it doesn't help.

And to give a positive example: though in mathematical circles Game Theory is considered to be von Neumann's baby, its successes are undoubtedly also due to the fact that it was developed together with a leading economist, Oscar Morgenstern. Speaking of Game Theory, I remember a good story, illustrating my last point, which appears in the book *A Beautiful Mind* [1]. Early in his career, mathematician John Nash thought he had solved the basic problems of General Relativity and, being at Princeton, he asked for an appointment with Albert Einstein, to tell him about it. He enthusiastically presented his results, lecturing the old sage for an hour or so. Einstein listened politely and then made just one comment: "Young man, I think it would profit you to learn some physics."

To sum up, Albert Einstein, again – 2005 is his year after all! --, this time talking to a mathematician: "My job is more difficult than yours; what you say has to be right, what I say also has to be true." Einstein means, of course, that what he says has to adequately describe an external reality, to be *true* a statement of physics has to tell us something about a world with independent existence, whereas for a mathematical statement being *right* only means that it follows the rules.

The proof of the extra-mathematical pudding shall always be in the eating.

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So, let us at get to the (potential) mathematization of narrative.

By our first truth, for mathematical tools or methods to promote the understanding of storytelling, they must be useful to narratology a field which is, at present, in a pre-Galilean phase. There are no mathematical results in it, no laws where some non-trivial truth can be arrived at through a formula, no valid predictions can be made, nothing significant that can be measured.

But speaking of measuring let me say that there are two ways, basically, in which a discipline can be mathematized. The first is more or less numerical: when people think of mathematization they usually think of numbers. And they have good reasons for that: until a few decades ago, almost all mathematization was about them. The first science where mathematics achieved great results, physics, is very numerical: given the velocity and the duration of movement, for example, we can calculate the distance an object has traveled, and so on, i.e. we get clear mathematical laws connect physical entities. And even in psychology, the discipline dealing with that most immaterial of entities, the human psyche, mathematics often contributes useful insights using numbers, though the truths arrived at in this case are statistical.

But what about stories? Well, interestingly there does seem a rather natural way to apply number to stories -- I recommend, as an introduction to it, John Allen Paulos's book *Once upon a number* [2]. In one sense, Paulos writes in his book, statistics and stories are diametrical opposites, statistics telling us "a little about a lot" and stories "a lot about a little". But in another way they are related: for storytelling, like statistics, explores the world of alternatives.

Think of it: a story where the protagonist has zero degree of freedom is not a very interesting story. A tale where the environment behaves in a totally predictable way is not much of a tale. For a narrative to be able to hold our interest for any length of time it must often make us wonder about what the

hero or heroine will do next. Plot-wise, a story is interesting because either: a) the outcome is unknown or, it is known, b) because the precise way of arriving at it is unknown. Factors like character or atmosphere or beautiful language apart, we keep turning the pages of a novel to see *what* will happen in the end or *how* it will happen, or both.

In T.S. Eliot's words:

What might have been is an abstraction
Remaining a perpetual possibility
Only in a world of speculation. [3]

But that "word of speculation" is precisely the world of the storyteller. A reader need not be aware of the unrealized possibilities in a narrative, but the footfalls down the passage the hero did not take still echo – to paraphrase the poet –, if not in memory then at least in his or her cognitive unconscious. The fact that Macbeth *may* or *may not* kill Duncan, is what makes his eventual killing of him truly terrible. Eliot uses the word "possibility". Substitute its near-synomym "probability" and you are in mathland.

The notion of "information measure" comes to mind, from Claude Shannon's theory. Dramatic events or possibilities can be described by their information value, a relationship can be established between how interesting a story is and how many, and of what likelihood, are the various alternatives of action open to its heroes, at every junction of their paths. Yet, again, it is difficult to see how such a calculus can be anything but a very basic reminder of truths that writer, reader and critics already know.

Numbers can only go a certain way in helping us understand the process of storytelling – not too far.

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And this leads us to the second way of mathematization, the way we shall call *algebraic*, that having to do with the manipulation of symbols, a way that did not become apparent before the twentieth century. And if mathematization by numerical techniques owes its first inspiration to physics, the algebraic starts with formal logic and then finds further ground in linguistics as also, later, the

area where the two overlap: the study of formal languages, a field also intimately related to computers.

Speaking of computers, let us go on a small detour.

We have been talking mostly about the scientific approach to a subject. But there is also the engineering approach. And perhaps this is more applicable here, as art, in a sense, resembles engineering more than it does science in its basic premises. For as in art, so in engineering, we set out to construct, not to interpret. And, come to think of it, in this engineering also resembles mathematics, for like it, it is axiomatic. Like mathematics, engineering creates its own fictional worlds, the worlds defined by its own axioms (the axioms of a washing machine, for example, the axioms of a carburetor) and within those universes, restricted by their own definitions, it can acquire great predictive power.

The aspect of modern engineering that is closer to storytelling is computer science and, more specifically, the field of Artificial Intelligence, whose aim is to construct simulations of human behavior. Storytelling is one of the human activities AI has tried to simulate, relying on algebraic techniques (call them “algorithmic”) since the creation of anything by computers has to be based on clear-cut principles and the rules of their combination.

Here are a few samples of stories generated by a program, in this case from Meehan’s TaleSpin, from the team of AI -- *and* narrative theory -- guru, Roger Schank:

Story number one:

One day Joe Bear was hungry. He asked his friend Irving Bird where some honey was. Irving told him there was a beehive in the oak tree. Joe threatened to hit Irving if he didn't tell him where some honey was. The End.

Story number two:

Joe Bear was hungry. He asked Irving Bird where some honey was. Irving refused to tell him, so Joe offered to bring him a worm if he'd tell him where some honey was. Irving agreed. But Joe didn't know where any worms were, so he asked Irving, who refused to say. So Joe offered to bring him a worm if

he'd tell him where a worm was. Irving agreed. But Joe didn't know where any worms were, so he asked Irving, who refused to say. The End.

Not exactly Dickens. But if these early samples be taken at all seriously – and they should be, for narratology should also study failed specimens – they fit quite well in a modernist program. If they sound more Ionesco than Ibsen, this is because the Theatre of the Absurd, like a lot of modernist writing, criticizes the contemporary world-view as that of a world losing its soul, i.e. depending more on form than on substance. This may be a highly negative criticism for human society. But it is nothing if not a compliment, for a formal theory.

But there is a more interesting way in which computers are aiding in the understanding of storytelling. For in fact, through computers, Eliot's "paths which we did not take" for the first time can become the very stuff of a fiction. In traditional fiction, the unrealized possibilities of an action are either stated as thoughts, transformed into oppressive emotions or, more often, remain unsaid, influencing the narrative by their unspoken-of existence in conceptual space, as lacunae, holes giving the bigger structure shape, as in certain sculptures of Henry Moore. But in a new genre, only made possible by computers, they become actual realities. In so-called *hypertext fiction* a hero can do both "A" and "not-A", though not at precisely the same time, since the reader can follow different, often contradictory, courses of action, within the same tale. This is made possible by the immateriality of the electronic medium, which like imagination, is fluid, not fixed by ink on paper. The linear sequence of events is not determined here by the author, who just provides the underlying structures. Assuming that some elements A, B, C, D, E, F, G form the basic conceptual diagram of a story, its graph of possibilities as it were, a classical version of a tale based on it might choose the path ACDF which may, in a certain universe of discourse, read as "boy meets girl, boy and girl get married, girl meets another boy, leaves first boy". Or, another, less classical alternative, may be ABEG, which may read "boy meets girl, boy and girl live together, boy meets another boy, first boy elopes with second boy" and so on. In old-style fiction, the author would choose one or the other. But in hypertext it is the reader that chooses, and everything is possible, ACDF,

ABEG, and many more, are available as different readings of the same fiction. Of course, what the reader – or rather the "user" -- of a hypertext fiction (Michael Joyce's *Afternoon* is the most famous example here) gains in possibilities, he or she probably loses in aesthetic pleasure. If you be handed an empty canvas and a set of oil colors and told you are at last totally free from the depressive hegemony of such people as Leonardo and Rembrandt and Manet and you can paint, for your living room wall, your own painting – is that a gain? To be told that *you*, the reader, can determine whether Raskolnikov will a) kill the old pawnbroker b) attempt to rehabilitate her or c) marry her, may liberate you from the imagination of Dostoyevsky. But is this freedom something to be hoped for?

Freedom is a great thing – in life. But enjoyment of art, narrative art more especially, is a field of human activity where we delight in having less, not more, choices.

It is no wonder then that hypertext fiction did not really catch on. Yet, hypertext has flourished, not by wearing the borrowed clothes of another art, but – like any robust new conceptual tool – by inventing its own. And I am here referring of course to the world of games, the new, complex computer games, where the player, or players, are given a complex environment in which to live – not 'read' or 'see' – his or her own adventures. But though the making of computer games requires the creation of a dense, underlying structures, that are visually and narratively rich, it is the choices made the user, the particular sequences of moves which are made every time a player plays a game, that constitute its *stories*. And it is in these choices, and their underlying grammar, the mysteries of storytelling lie.

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Before I get to the most promising, to my mind, avenue open to the future mathematizers of narrative, let me go back to the father of narratology.

Aristotle, author of the *Poetics*, was also the father of most sciences – also, famously, of logic. And his great talent, quite similar in structure to Euclid's, was that of a collector and taxonomist. For it is taxonomy, the idea of criteria, which first puts order on the chaotic material of reality, any reality,

reducing it to well-defined classes, a set of sets ordered by relationships of inclusion, a graph of some sort.

We said earlier that to be mathematizable, a science has to reach a certain level of maturity. And maturity for a science, as for an individual, means among other things the abandonment of the juvenile, narcissistic fantasies of omnipotence. All sciences start out by dreaming of a Theory of Everything. But all begin to advance when they moderate their demands and accept that progress has to be made piecemeal, step-by-arduous-step.

So, it comes as no surprise that the first person to achieve something important in the mathematization of narrative was a folklorist, a man used to ordering masses of material by certain criteria. He did not start by trying to construct some grand-scheme, but to classify the particular kind of tales he was collecting. (Let me say here that there is no hierarchically constructed taxonomy of tales, no Linnaeus of folklore. There is of course the Index of Aarne and Thompson [4] -- usually referred to by the authors' names, whereby a certain fairytale may be of type "AT 432" -- but it is a totally unstructured thing, just a long list really.) But Propp was a follower of the new, back then, before WWII, Russian Formalist School and obviously had the right attitude. He isolated a particular class of tales from the Russian tradition, the "magical folktales", i.e. the tales -- also prominent in other traditions -- in which the hero embarks on a journey of adventure, to achieve a certain difficult goal. ("Jack and the Beanstalk" is a good example from the western tradition.) And studying these, he found a common structure so clear that what he describes in his seminal paper, "The Morphology of the Russian Magical Folktale" [5], can be called mathematical.

To find in his object of study rules that were clear enough to merit the name of being formal (algebraic) Propp did what any good scientist would do: he restricted his field of enquiry. Science is a constant struggle between centrifugal and centripetal forces, the centrifugal pulling us towards generalization, big, general laws, increased understanding and the centripetal towards specificity, clear and strong results in particular sectors. And Propp was brilliant enough, or lucky enough, to locate an ideal field of enquiry,

narrow enough to be clear and formalizable, and general enough to be interesting outside a narrow band of specialists.

By restricting his attention to a certain kind of story, a sub-genre, Propp was able to locate the “atoms” of the stories in question, if you want, which combined give all else. These he called, rather confusingly I think, “functions” and he defined some basic rules by which they are ordered and combined, a process we know from modern algebra, and also similar to work in the study of formal languages, a theory which shares, via Chomsky, its ancestry with the Russian Formalists. Propp created a basic formalism by which a magical tale can be described as a series of his “functions”, some of which are *constant* and omnipresent (e.g. the tale always starts when something causes a state of equilibrium in the hero’s place of residence and he leaves it in order to solve a problem that will put it back into order; and there is always a “magical helper”, i.e. a being with magical qualities to assist the hero) and some are *variable*, as for example the number of feats the hero must perform in order to achieve a certain goal, or the routine of some sub-goals, the sub-routines we might call them, which describe elements of his basic plot.

Propp’s study remained an isolated case for many decades, marginalized by language and culture. But then, the American mythologist Joseph Campbell constructed something similar in his *Hero with a Thousand Faces* [6], what he called the “monomyth”, i.e. a basic structure with “functions”, some constant and some variable, which can describe the basic, central myth of many – if not all – traditions, what Campbell called the Journey of the Hero.

Again, there was a lull of some decades – Propp interested only folklorists and Campbell only mythologists, and then suddenly there is a new explosion. A very strong new school of studies has appeared, made up of people who speak an unusually exact language. However, this did not happen in some academic haven, through articles in a learned journal or scientific conferences, but in the most materialist – and at the same time most illusionist -- of places, i.e. Hollywood.

It is not surprising, really. Think of Archimedes and the tyrant of Syracuse. Think of physicists and the bomb. Think of medicine and the pharmaceutical multi-nationals. Turing and the War effort. Boole and IBM. We don't like it, but it's a fact of life: science and industry, knowledge and money, live in non-Euclidean geometries, their lifelines, although parallel, often intersect – too often. Einstein and Truman. Vladimir Propp and Stephen Spielberg.

It does make sense: the spirit behind engineering is the practical mentality of commerce. A scientist wants the truth. An engineer wants effectiveness. And he is paid to find it by a business, that wants to make money. So, it is no wonder that Hollywood would at some point spearhead the research into storytelling, with very practical aims: to find a foolproof way of creating a good story, quickly and cheaply – or, if that is too much to ask, to find some "objective" criteria by which it can be judged, criteria that producers, people who only know how to read spreadsheets, can understand, clear and simple. Aristotle was immediately commissioned and acknowledged to be the guru of all storytellers. And then two parallel strands developed, both very systematic and increasingly formalist in their approach: the one studied character and its relationship with action, with its origin in Stanislavski (with a solid beginning in Aristotle) and the very exact elaboration and formalization of his ideas by Lajos Egri's in the book *Art of Dramatic Writing* [7]. And the other worked on plot, with heavy debts to both Propp and Campbell, and a whiff of Carl Jung. The two strands started to come together in Syd Field's *Screenplay* [8] and Robert McKee's *Story* [9] the gospels of modern Hollywood writing. These books attempt to describe an archetypal, ideal script, often reminding us of course that art is all about imagination, originality and invention, but warning that their system is the only one, really, and underlying the most diverse films, from *Ben Hur* to *L' Avventura*, from *ET* to *Yojimbo.*

The authors of these books, and even more their readers, would be surprised to find themselves mentioned in a meeting including the word 'mathematics' in its title. But any mathematical reader will immediately see

that in them is being developed an axiomatic and formal, diagrams and all, theory of storytelling, the Theory of Everything that narratology dreams of.

Some of the basic axioms of this Hollywood cosmology of film are:

1. All stories are really quest stories.

1.1 If a story does not appear to be a quest story at first glance, delve deeper into it and unearth the quest story within it.

1.2 If you fail in 1.1 drop the story, it ain't worth it.

2. All quest stories are about a sympathetic hero searching for, and finding, a treasure.

2.1 The treasure may be material (money, object, secret weapon, world cup) or immaterial (love, salvation, knowledge, etc.)

3. The interest of the story is determined by certain factors, among them which: a) how important is the 'treasure', b) how bad the hero/heroine want it, c) how difficult the quest is. The more difficult, the more tickets at the box office.

4. The difficulty of the journey is incarnated in a person, called the Antagonist.

And so on.

Obviously, serious people know that all these generalizations are, like all generalizations, too extreme. The pattern is too clean for comfort, one immediately suspects that a) it is probably often wrong or, b) even if it is right and we all start following it, this will mean the end of movies as we know them. But, of course, a *great* percentage of Hollywood films is created using these principles, guiding writers, producers, as well as the 'screen doctors' – yes, such a specialty exists! – who are called in to examine faulty product and to suggest remedies.

Watch the last ten, say, big animated feature length films, produced by the big studios. Examine them for patterns, *à la* Propp. You will find an exact model that fits Campbell like a glove, not necessarily the original Campbell but

the Hollywood one, as exemplified in one of the mega-sellers in the screenwriting world, *The Writer's Journey* [10].

I am not a Hollywood producer, and I do not particularly care for the future of the big American film industry. I have an artist's reflexes and admire Mizoguchi more than Cecil B. DeMille, Bergman more than Spielberg. To live in a world where films would be made by some story-generation program is not — I repeat not! – my image of utopia.

As a writer of fictions I rebel, every cell in my body rebels. When, as an audience member, I detect the invisible – to the *hoi polloi* – hand of the script doctor guiding the plot, *d' après* Syd Field, I get angry. But as a writer who started life with mathematics, I cannot help but feel that these guys have caught on to something interesting. And it does not surprise me, that what is at some level a very long, very elaborate and very costly series of experiments to try and reduce all writing to some formulas – and don't forget please that we do call such films "formulaic" -- should come from the business world and not from academia. It's not the first time something of this kind happens!

I think that this new "theory" of screenwriting developed in Hollywood, partly through its indebtedness to some great scholars and partly through the pressures of the market, shows us opens up a way in which to study storytelling with mathematical means, with the aim of understanding it better. The showbiz yuppies, the screenwriters and the script-doctors and the script coaches, the writers of books with titles like *How Aristotle will help you write a blockbuster* have created a model which is ready to fertilize the world of critical thinking about storytelling. Not all stories can be reduced to quests – but many can. And this is all about mathematization. Quests are known to be particularly friendly to mathematics, there is a lot of mathematics to explore them with, from graph theory to algorithmic information theory to game theory. And the algebraic approach needed to work with the laws of combination is there.

Human culture does not progress solely through the work of the angels, *les bons sentiments font de la mauvaise littérature* and we can find treasure in the unlikeliest of places.

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